

Spectral Unmixing In HSI: A Case Study

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Abstract:

In HSI, the unmixing of images plays an imperative role since the initial years. The collection of spectral signatures from the existing environments is perpetually its fusion of several constituents originated in the spatial range. HSI obtains a three-dimensional dataset called hypercube, having one spectral and two spatial dimensions. The exploration of HSI is based on the spectral decay of the pixels through the spectral unmixing method, having applications in detection of the target, unsupervised segmentation of the image, etc. The perilous part is to define the endmembers used as the references for the process of unmixing. Hence, inclusive details of the unmixing method are required as its application is extensively growing. The unmixing methods are summarized in three categories: extraction of endmember, selection of endmember combinations, and abundance estimation. The paper consequently provides an outline of HSI unmixing and its applications. The primary objective is to provide a historical outline of the popular methods of unmixing and to discuss certain popular techniques in detail. In HSI unmixing, LMM is the dominant archetypal besides it is a foundation of a huge diversity of unmixing methods in HSI, hence a prominent part of the review embraces it. Furthermore, in the HSI unmixing, nonlinearity is a critical factor in real-world situations. While numerous models for nonlinear unmixing are projected, in recent times there occurred an explosion of nonlinear models for unmixing. Henceforth, we deliver an outline of some recent expansions in the nonlinear unmixing literature. The objective of the paper is threefold: an overview for new researchers and for those already functioning and searching for literature in this arena. The quantitative measure of certain existing methods helps to analyze current progress and to anticipate imminent growth. Lastly, the review is structured according to the elementary computational method of unmixing: geometrical, statistical, and heuristic approaches with a small segment of statistical versus geometrical methods.

Keywords: hyperspectral images (HSI), hyperspectral unmixing (HU), neural network (NN), reconstruction error (RE), endmember determination (ED), and dimension reduction (DR).

INTRODUCTION

HU is defined as the decomposition of the restrained spectrum for an assorted pixel into a set that has pure spectral signatures termed as endmembers with its corresponding abundances that indicates the fractional region coverage of every endmember existing in the pixel [1]. For given mixed pixels, the need is to distinguish the distinct essential materials existent in the mixture,

plus the amounts of its appearance. For a scene, the endmembers usually correspond to similar macroscopic objects. HU is the ability to provide valuable subpixel detail in various tactical scenarios. In the arena of HSI processing, HU is the calculation of the insignificant input of the endmembers since it establishes the vertices of a convex polytope casing the data points in high dimensionality for the image. In remotely sensed imagery, numerous studies exploit the spectral information inherent to evaluate mixtures of diverse materials, considering every pixel to be liberated of its spatial neighbours. Recently, spatial information is adapted in the growth of HU approaches. It is validated that the spatial and spectral information together leads to enhancements in the HU results [2].

Several research communities established various unmixing algorithms to use HSI data to resolve certain problems. The scientists advanced the unmixing task with the outlook of scrupulous models that prudently perceives the interactions of light and mixed material. Despite their accurateness, the models remained unable to deliver the numerical inconsistency intrinsic in remote sensing interpretations, and the outcomes of unmixing, whereas precise for the situation defined, there is a deficiency of preferred robustness for overall implementation. On the contrary, engineers and mathematicians frequently avoid the physics in favor of modest, manageable descriptions to exploit vigorous geometric models, to attain certain optimality. The statistical modeling, unfortunately, frequently fails to attain a great degree of physical aspect that guarantees accuracy with substantially probable solutions for distinct pixels. The main motive to develop a taxonomy is to filter to the least possible set, the algorithm universe, risen from distinct communities. To execute this purification, a method of hierarchical unifying algorithms is developed by the beliefs that employ three significant features of processing. Every sequential characteristic is a filter of accumulative granularity and the unification of the norms delivering superior enhancement in distinctive procedures. The HU progression is truly a concatenation of three individual events, all with distinctive purposes. For every processing phase, the equivalent taxonomic methodology is applied to expose methods that are common across the three algorithm types. When output assessments are required, a catalogue rationalizes the complex practice of relative outcome exploration by naturally separating clusters of procedures that entail analogous inputs or common computational units. Furthermore, the mission of experts is to design a model for definite applications, such as unmixing, in an inhibited operative environment with restricted information and computational resources. For the algorithms, the taxonomies organize information to facilitate the analysis.

1. Phases of Unmixing

For HU, algorithms use diverse scientific procedures that determine the endmembers and their abundances. Since HSI possesses huge volumes of data, certain unmixing algorithms initially reduce the data dimension to decrease the equivalent computation. However, signifying data in a compact aspect is a reduction to the precision in the anticipated solicitation product. The HU delinquent is decomposed into the structure of three successive phases: dimension reduction, determination of endmember, and inversion. In the first stage, the elements of the data are reduced. This phase is elective and is entreated merely by few procedures to decrease the computational burden of successive processing. In the second phase, the endmembers are estimated from their constituent the mixed pixels. Finally, the last stage produces the

abundance planes allowing the approximation of the minuscule abundances for every assorted pixel. Several methodologies are defined in the literature to accomplish the tasks at the respective stage.

1.1. Organization Structure

First, the unmixing problem is reframed as a distinctive instance of the comprehensive inverse delinquent [3] and then elementary queries are modeled concerning the intrinsic philosophical perceptions employed by every algorithm. The evaluation emphasizes the major norms that motivate the procedures. For a single comprehensive class of techniques, the descendant flow of the taxonomy, from a hierarchical view, shares a mutual aim to discriminate the variability of methods with growing granularity that occurs along pathways. With no probabilistic model, the algorithms adopt a Gaussian model for the uncertainty of the data. The Gaussian techniques are categorized in a more exhaustive distinction i.e. maximum likelihood and maximum a posteriori. The top-down analysis of the unmixing HSI provides three standards to classify the unmixing algorithms:

- Data interpretation, to specify the interpretation of mixed-pixel spectra by an algorithm.
- Randomness description determines the way an algorithm integrates the uncertainty of the data.
- A criterion of optimization, indicating the objective function being elevated.

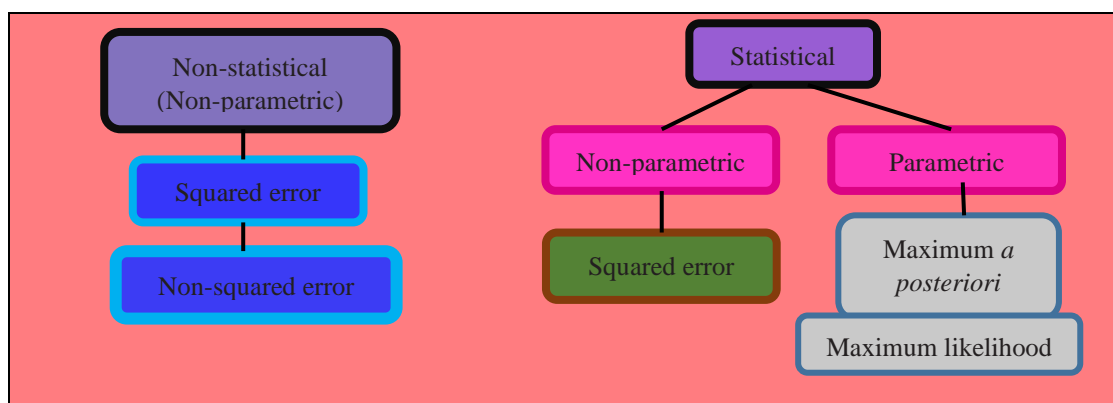


Fig.1. Illustrative organization of the classification criteria in HU [4].

A procedure deduces the data in twofold ways. If the mixed pixels are processes via statistical measures e.g., means or covariances, the procedure is statistical. Subtly, statistical procedures familiarise the cumulative conduct quantitatively for huge data to process an individual pixel and have no information its probabilistic nature. The figure 1 illustrates the organization of the taxonomy standards in HU. The dimension reduction achieves a basis transformation (linear transforms) frequently resulting from the data covariance deprived of a data probability distribution. Essentially, the pixel dimension is compact by exhausting facts gained from a huge data set. Procedures lacking statistical dealings are non-statistical. This discrepancy is particularly significant in target detection, where statistical categorizations of non-target behavior obscure the discovery of low-probability targets. Similarly, an algorithm depends on

the data analysis that directly reflects how its fundamental randomness is addressed. If the objective function is optimized, the algorithms are deemed optimal. Parametric algorithms enhance certain density combinations. Non-parametric procedures exploit the cost functions multitude, nevertheless, the predominant optimization norm is squared error minimization.

Every criterion defines a vital trait of relating the process to the data, and since the norms are not explicit to any specific process, the classified apportioning atlases from one process to another. The three classification standards distinguish procedures by an approach, auxiliary variation within every session algorithm is attained by a task-specific feature set. In contrast to the criteria that apply to each phase of unmixing, the approach recognizes borders for defining processes on operating features, distinctive at each stage.

2. Dimension-Reduction

This fragment delivers an assessment of algorithms used to reduce the HSI dimension. Dimension-reduction procedures do not reduce the data dimension to recreate an estimate of the unique data. Element reduction aims to attain a nominal illustration of the signal in a space of lower-dimensionality to preserve the vital information adequately for efficacious unmixing in the inferior aspect. Preferably, element drop procedures are intended to perform unmixing procedures in the lower dimension.

2.1. Dimension-Reduction Algorithm

Concerning the data, the algorithms in this section do not deduce any probability density function, and hence are nonparametric. The statistical process transformations arise from their numerical information approximately the data and are distinguished by the optimization norm. Based on the squared error, the principal-component analysis (PCA) [4] technique categorizes orthogonal axes to reduce dimensionality by the execution of an Eigen putrefaction of a covariance evaluation. An alternate statistical technique to optimize SNR is maximum noise fraction (MNF) [5]. For dimension reduction, a non-statistical technique to optimize squared error is the ORASIS that designs a sequence of HSI handling modules to accomplish data from diverse HU platforms. In a scene, to achieve DR it is essential to recognize a subgroup of evocative, or ideal pixels to deliver the inconsistency in it. For a prospect, a new pixel is gathered by the sensor and equated to every ideal pixel through an angle metric. For every adequate variation shown by the new pixels from the existing exemplars, adds it to the exemplar set. In the absence of adequate variation, the exemplar set is unaltered. From the current set of models, an orthogonal base is created sporadically by a reformed Gram-Schmidt course to add novel extents till each pattern is approximated in a prescribed tolerance. The exemplars, in the reduced dimension, are acquiesced to another module to define endmembers geometrically. The capability of the DR process is associated with the importance of the user-defined pre screen error-angle threshold, restricting the admittance of new exemplars.

3. Endmember-Determination

This fragment addresses ED, which is the principal production of unmixing. The purpose of this phase is the estimation of the integral spectra. However, the elements must be non-negative to have physical attainability. Furthermore, ED must preserve somatic features of the essential

element. So, creating an ED process that satisfies both somatic and scientific necessities is a significant task, making ED the demanding fragment of the unmixing. ED is meticulously allied with the substantial identification proficiencies of HU. A precise evaluation of subpixel alignment initiates with a consistent approximation of untainted substances that encompass assorted pixels in the scene. The endmember determination method does not only cope with mining physically significant and identifiable spectra but is required to perform in situations with restricted and imperfect information.

For a given scene, the endmember set is assumed to be invariant by most of the spectral unmixing approaches. The multiple endmember spectra are used to symbolize the same class in a more realistic scenario, owing to endmember inconsistency arising from varying illumination situations and physical alterations in generally defined endmember classes [5]. Furthermore, for a single pixel, the actual quantity of endmembers enclosed is typically lesser compared to the number of endmembers existing across the complete scene [6]. Several spectral-only methods are established to address these two issues that permit the quantity and category of endmember and its equivalent spectral signatures to differ on a per-pixel basis [6]. However, these approaches frequently continue the confusion of spectral and computational intricacy persuaded by several endmember arrangements. Similar endmember combinations are considered to be shared by the nearby pixels, which includes spatial information to select the endmember combinations, hence, lighten confusion of spectra and increase computational efficacy. To select the endmember combinations, the spatial-spectral methods are categorized as per-pixel (PP) and per-field (PF) methods. Across an image, where the pure pixels or endmembers are extensively dispersed, a PP scheme is typically useful to attain an endmember set, for an assorted pixel spatially near to the pixel. Spectrally untainted pixels are recognized by controlled classification with severe measures [7]; manual analysis, supreme abundances of global endmembers, thresholding of histogram or algorithms determining the determination.

For an assorted pixel situated in the middle of predetermined spatial environs, the optimal PP endmember set is designated by a reiteration unmixing process like MESMA [7], an endmember scoring outline, or an endmember fusion procedure through spatial averaging. For synthesis, a further refined three-dimensional interpolator is used to produce the vegetation indices of bare soil and complete green vegetation shield, and yield more precise evaluations of fractional green vegetation shelter compared to the inverse distance weighting (IDW). For coarse-resolution images, along with interpolation, extrapolation is used for the synthesis of endmember spectra where pure pixels are lacking [8]. The multivariate linear regression prototypes are created for training samples when the endmember fractions are known, via the sections as liberated variables and the sample pixels spectra as dependent variables. For each endmember, the spectra are produced by inducing the applicable regression archetypal to the cover section. The locally standardized regression statistics are used to integrate endmember inconsistency that comprises mean and variance matrices for constructing regression prototypes. To develop synthetic endmember spectra, in [8], the geographically weighted regression (GWR) is adopted. Both approaches can yield spatially flexible endmember signatures. Altogether the aforementioned PP approaches symbolize endmembers as a distinct set. Nevertheless, endmembers are similarly signified by a continuous distribution.

To select the endmember combinations, the per-field methods typically partition an assorted scene into a consistent field, assigning every arena a trivial set of endmembers. The image partitions are usually assembled to form consistent fields to signify diverse land-cover forms rendering to earlier facts of the scene. In every field, the endmembers are either selected by the manual intervention or automatic algorithm for endmember extraction. In HSI, the pre processing chain is used to acquire the qualified bifacial reflectance, demarcated while the quantity of emission perceived in a specified way, compared to the quantity of emission received by a correlated root of the light. It is generally proficient by isolating the sedate luminosity sustained by an ideally Lambertian calibration panel in an identical computing setup. An outline of the diverse definitions and analyses of reflectances and albedos can be found in [9]. For HU, one usually contemplates the endmembers to be an identical deposit of spectra for the intact image, while the profusions will differ on a PP basis. Numerous methods then exist: for the known endmembers, unmixing converges to the inversion of mixing archetypal. For the unknown endmembers, they are obtained by spectral data. The HU approaches mostly engage an endmember extraction algorithm for its discovery, subsequently comes the inversion step, however certain algorithms simultaneously discover the endmembers plus abundances.

For the remotely sensed image, the endmember extraction technique aims to define endmember spectra by itself, since the endmembers of the image giving the identical spatial measure and distinctive settings as the image for HU. According to the convex geometry theory, for a given spectral space, numerous algorithms attempt to explore the pixel set. For remotely sensed images, the geometry-based algorithms exploit their spectral properties and incline to be vulnerable to noise and deviated pixels. The spatial data is fused in the extraction of endmembers leads to the expansion of spatially aligned processes, the enhancement, and the growth of pre processing components. Spatially aligned procedures remain a primarily diverse method to commutative centered procedures – the prior relies on perceiving untainted pixels or regions in the spatial perspective, instead of examining the spectral excessiveness of pixels. The advanced spectral-only procedures use spatial data to increase the performance of commutative-based processes. The pre processing approaches preceding the abstraction of endmembers are exclusively extensible since the alteration of prevailing endmember extraction processes is not desired for the pre processing methods.

The algorithms for mining endmember that are spatially oriented, deliver a method to evaluate the pureness of a pixel or a spatial region devoid of utilizing the commutative processes [10]. In [10] an SPEE algorithm is established to identify untainted spatial regions. The purity index for the spatial region is defined by either an intensity dimension or feature degree measurement. To acquire endmember candidates, a threshold is applied. For remotely sensed imagery [11], a graph-based three-dimensional modification procedure is assumed to decrease the number of endmember candidates. The pure spatial region is acknowledged using the multi-scale representation. With the multi-grid structure, a sequence of leveled images is created and endmember ranges are mined. The leveled spectra are equal to the regular spectrally alike and spatially contiguous pixels. To enhance spectral-only algorithms, an assortment of processes for endmember abstraction is proposed to integrate spatial information. The spectrally analogous and spatially contiguous endmember candidates created by spectral processes are

either averaged or the endmember candidates existing in the consistent regions are selected. In every partitioned subset of the image, endmember candidates acknowledged by PPI are assessed rendering to local homogeneous measures to attain an endmember deposit. Before the extraction of endmembers, the fusion of spatial data in algorithms for endmembers abstraction, in contrast, reflects spatial data through a dispensation stage. With any endmember abstraction procedure, the pre processing unit is united. For the spatial pre processing (SPP) algorithm [12], on a PP basis, a spatially derivative increment factor modifies the original spectral signature to facilitate pixels situated in spatially consistent areas that involve a lesser amount of alterations compared to spatially assorted regions. Intrinsically, applying a commutative process to the attuned image, pixels situated in spatially consistent areas are acknowledged as endmembers. For remotely sensed imagery, the segregation is done with numerous pre processing methods that adopt segmentation or clustering methods and for the endmember extraction algorithms, the mean spectra of every region are used. At the pre processing level, the spatial-spectral pre processing (SSPP) algorithm [12] delivers a further unified outline to syndicate both three-dimensional consistency and spectral pureness. Within every region created by unsupervised clustering, the identification of a subclass of spatially consistent and spectrally untainted pixels is done. Spatial plus spectral data are united to improve local neighborhood weights. Through the weighted thresholding, a sight is distributed into consistent and evolution areas. The algorithms for endmember extraction considered the pixels that collapse in the consistent areas. Recently, numerous pre processing approaches have been advanced to decrease the extent of the inventive data set although, the precision of endmember abstraction is retained. For endmember extraction, a probable disadvantage of spatial-spectral algorithms is their negligence to anomalous endmembers [13]. This problem can be overcome by integrating the anomaly detection techniques with spatial-spectral processes to excerpt consistent as well as irregular endmembers.

3.1. Nonlinear Unmixing

Though HU algorithms established on LMM are developed as well as flourished, the nonlinear HU processes nonetheless prove that for a scene, the physical mechanisms are proficiently modeled performing unmixing reliably.

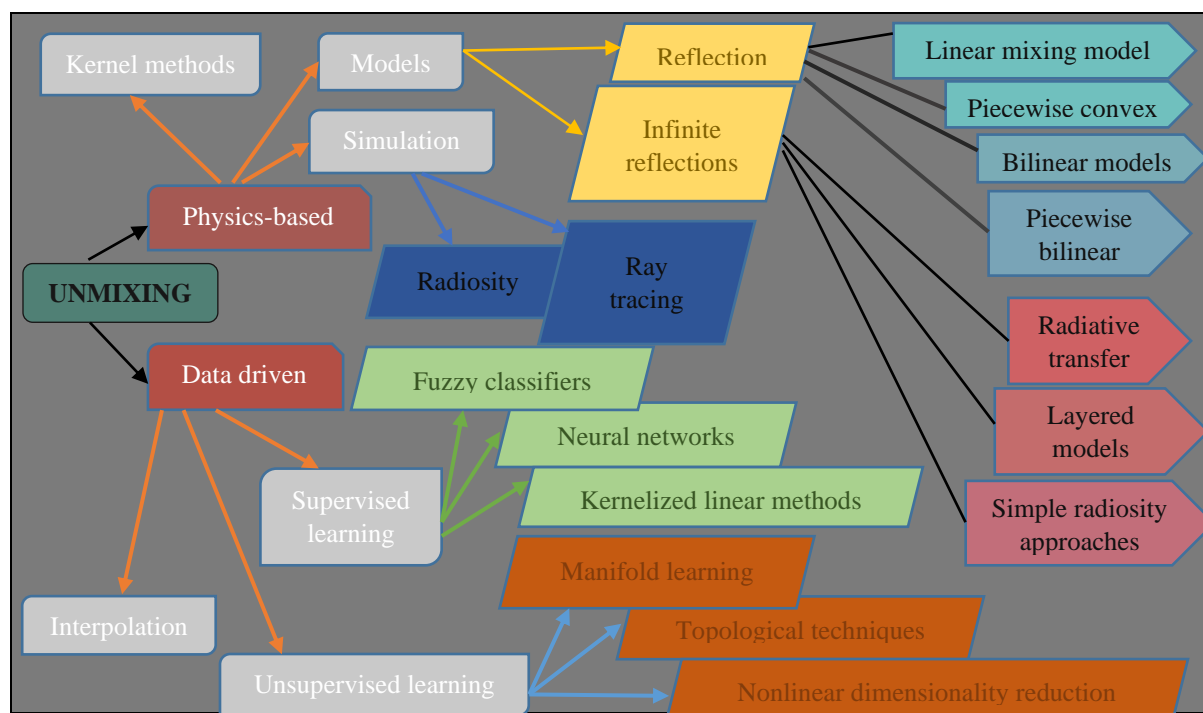


Fig. 2: Categorization of the different unmixing techniques [13]

However, enormous distinct techniques are recognized. Several different types of techniques for nonlinear HU are recognized, and a classification is presented in figure 2. The main level is a discrepancy prepared amongst physics and data-driven methods where no core physical norms are needed. The physics-based procedures explicitly alienated the model of the spectral mixing effects that vary in degrees of intricacy. In its simulation methods, the radiance interaction with sight is modeled through computer visuals and kernel procedures. Unlike LMM, which needs physical specification for critical docility, physical prototypes are not dependent on simple mathematical solutions. Despite the complexity of these solutions, scientists endure effective means to perform unmixing.

3.1.1. Physics-based approaches

3.1.1.1. Mixing models

These models are classified by the considered quantity of reflections. Distinct reflection models are typically leading to the LMM. As the light rays undergo two reflections, secondary illumination is introduced to derive the bilinear models. Largely, bilinear prototypes are reflected in the prototypes that embrace a great number of endmembers with practical secondary consideration. They usually yield improved spectral reconstructions compared to LMM, though they suffer from a variety of other complications, for instance excessively free variables, overfitting, collinearity of virtuality, and invalid equivalence of archetypal enforced constraints that are non-realistic substantially. Linear and bilinear prototypes, both are piecewise methods, partitioning the data customary in diverse segments modeling them individually

for both. For piece-wise methods typically altogether the endmembers are not extant in every pixel, with fixed subclasses of endmembers come across in assortments.

Lastly, numerous methods contemplate an endless quantity of reflections. Layered approaches are reflected as physical prototypes that ruminate an unlimited quantity of illuminations too. For instance, further methods where layered prototypes are engaged, where the scattering interfaces at the border, model every particle as the illumination propagated at the border among two media of diverse optical coefficients. Among the multiple layered interfaces, several interactions can be modeled as reflections/refraction. The physics-based kernels are used, where a kernel function is derived, and used in a kernelized form of a linear processing restraint. This method is reflected in a physics-based machine learning technique.

3.1.1.1.1. Piece-wise linear methods

For HU, a stepwise regression tree technique is projected [14]. It begins with the optimum linearly unmixed product, and then the data customary is divided into twofold subgroups, and the variance in RE is calculated for these linearly unmixed data sets distinctly. It then determines the optimum split, besides if parting the data set decreases the RE beyond specific threshold assessment, and the subgroups are superior to an agreed minutest, the data is riven into binary nodes. The procedure is recurrently aimed at the nodes in a regression hierarchy pattern, which ultimately leads to a piecewise linear estimate of the intricate nonlinear association among profusions and spectral statistics. The method is formerly used to estimate the subpixel forestry profusion. For a scene, these approaches determine the number of convex regions, which are created on the reflection that in the scenario of real-world unmixing, simply the blends of definite subsets of endmembers occurs, that often discover mixtures of endmembers devouring substantial contributions from all endmembers. In a real-world image, for instance, using 5 endmembers (soil, metal, grass, trees, concrete), mixing is presented by the grass endmember by soil and tree endmember whereas the tangible endmember is merely combined with the metal and grass. None of the pixels will comprise instances of every endmember, besides the statistics can be defined impeccably by using 3 endmembers. The data from a geometrical view will be primarily placed on definite faces or sub-simplices of the endmember simplex, plus an improved data depiction is attained about numerous lower-dimensional simplices, each labeling a subset of endmembers showing mixing. This gives a piecewise convex depiction of data with extremely composite data cloud, and occasionally radical enhancements to select endmember and in unmixing results. An additional benefit of the approaches is that endmembers are established in the simplex of alternate customary of endmembers ensuing an overlying endmember array. Sparse unmixing methods provide a spectral archive and attempt to discover an optimum lined illustration of the facts, however with added restraints that limit the number of endmembers having a nonzero profusion. The aim to limit the number of endmembers in every pixel are huge libraries employed, the number of endmembers constrained to evade overfitting or to retain the problem tractability. The mixtures with a high contribution by endmembers are evaded since they are impractical in firm

scenarios, or may encourage the usage of a similar set of library endmembers for the whole image.

3.1.1.1.2. Linear mixing model (LMM)

While the portrayal of the HU problem is wide-ranging, certain properties can be undertaken in further aspects as the LMM, being it a simple, very widespread mixing model. Considering the geometrical analysis of the LMM [15], the pixel created by the LMM lacking noise will be present in the curved hull traversed by the endmembers. Subsequently one generally undertakes that the endmembers quantity is greatly less than the number of spectral bands. Numerous techniques for resolving the LMM exist. In [16] a universal outline of the HU problem, and recent and widespread overviews of linear unmixing techniques are given.

3.1.1.2. Simulation techniques

For computer graphics, numerous methods for the recreation of simulated extracts are protracted to produce virtual HSI. In the scene, for each material, ray tracing requires an exhaustive model of the optical characteristics. To model a virtual scene in a high aspect, the computational necessities are huge. For ray tracing the relations between reflectance and abundances cannot be inverted easily, and are therefore challenging to be used in practical applications. For simulation, another approach for virtual scenes is the radiosity, resultant of the execution equation by introducing numerous vulgarisations that leads to computationally additional docile technique, however, several problems faced in ray outlining still apparent themselves. The unique benefit of radiosity is it's certain modest situations in which precise analytical results can be designed.

3.1.1.2.1. Bilinear interactions

The LMM mechanism is well providing the dissimilar components of the pixel are spatially isolated regions with no interfaces between them [16]. For instance, the spatial structure is less trivial with rocks or vegetation that strikes the surface, illuminations would endure several reflections before it reaches the sensor. A simple model to the manifold illuminations is to ponder merely bilinear interfaces, with light rays interacting two dissimilar materials. Bilinear interface indicates that an entity is not illuminated by the source of light, but the light from another object. For the intensity of an assumed spectral ensemble and the profusion of the flora endmember, it is observed that they have nonlinearity, and are strongly dependent on the soil form. Even nearby 100% herbal coverage, the soil form impacts the perceived spectra owing to NIR illumination that penetrates the canopy and interacts with the soil. This interaction, therefore, needs to be justified, and a bilinear model [16] is, proposed incorporating this interaction with the conclusion that ominously minor reconstruction and abundance errors are obtained with the bilinear model than that obtained by the LMM with these data sets. Though the model permits improved reconstructions than the LMM, the abundances frequently display higher unorthodoxy to the ground truth than the lined ones. It is claimed since the bilinear

process has fairly huge abundances, alternate analysis is required for abundances to deal with bilinear models.

3.1.2. Data-driven approaches

For nonlinear unmixing methods, another large family of is built on data-driven measures. For nonlinear unmixing, all supervised techniques are precisely appropriate. MLPs is an NN architecture to acquire nonlinear associations in data sets, however, it needs archetypal training data that is cumbersome to attain in many HU applications. A recent example of NN architectures is the usage of self-connecting NN to find a sparse data representation that is inferred as a nonlinear HU outcome. This NN design fits substantially to the unsupervised methods, as it simply entails the data itself with no added information. The linear model's algorithms are usually altered to deal with nonlinear models by using the kernels. In HU, the method is widely incorporated by numerous researchers to generate nonlinear forms of linear HU methods. Kernel methods still depend on the selection of the kernel utility strongly. Maximum unsupervised methods represent the nonlinear manifold comprising the data points in a certain way. Additional methods directly function on the data manifold, as it represents the data as a customary of related reserves beside the manifold, and employs a distance-based HU method on these expenses. The benefit of these methods is their fully unsupervised nature without manual intervention.

3.1.2.1. Unsupervised Hyperspectral Unmixing

For the supervised setting, the endmember is known earlier; merely the abundance is estimated. Though the task of HU is streamlined in this setting, it is commonly obstinate to attain realistic endmembers, consequently, hindering the attainment of virtuous HU approximations. Thus, the weakly supervised techniques [16] were projected. An enormous library of quantifiable spectra is collected by a field spectrometer beforehand. Formerly, the HU's job is to find an optimum subset of material spectra in the library to best represent all the pixels in the HSI plus their abundance maps. However, the library is different from optimal for the spectra in it are not standardly unified. First, for diverse HSI sensors, the spectral signatures of the identical material can be very erratic. Secondly, for the HSI recorded by different sensors, both the number of spectral bands and the electromagnetic range of recorded spectra are chiefly altered as well. Lastly, the recording situations are dissimilar—some HSI is taken from outer space, while others are obtained from the airplane or even in the lab. The atmospheric effects create diverse recording situations resulting in diverse spectral appearances. Briefly, the flaws of the spectral library pass side effects into this kind of approach. Generally, the endmembers are learned from the HSI itself to guarantee spectral coherence. The unsupervised HU approaches are chosen, where the endmember and abundances are mutually learned from the HSI.

3.1.2.2. Kernel Methods

3.1.2.2.1. Kernelized linear unmixing algorithms (The kernel trick)

It is a popular process to introduce nonlinearities in a linear process. The key indicator is that plotting occurs amongst the data trait and a rich dimensional feature trait so that the delinquent develops linearity in this feature space. Since this feature trait is not desired to be constructed explicitly, the kernel trick is applied, based on these twofold:

- The algorithm is written in terms of an inner product that is to be used in the featured trait.
- In the featured trait, a kernel function outlines the inner product between twofold points as a purpose of the two equivalent facts in the data trait.

An orthogonal subspace projection (OSP) process is adapted in [17] with the kernel-OSP algorithm. It can be used for nonlinear EE or detection of the target. Validation is provided on synthetic and physical data, with the use of the Gaussian RBF kernel, where the KOSP overtakes OSP on a task of subpixel target recognition. A kernelized entirely inhibited least-squares process is a resultant that allows one to accomplish FCLSU in the featured trait rather than the data trait, by any kernel function.

3.2. General Framework based HSI Unmixing Classification

Present methodologies attempt to persuade the endmembers from the illustrative data. They either attempt to select certain illustrative pixel spectra as the finest estimate to the endmembers [17] or computing estimates for the transformations of the data in illustrative.

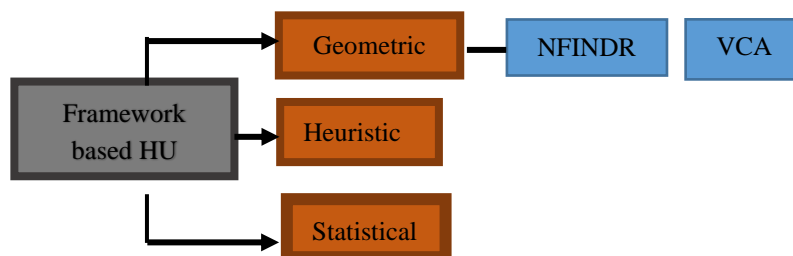


Fig. 3. Framework based hyperspectral unmixing [17]

The figure 3 depicts the framework that forms the basis of HSI unmixing. The latter is the principal technique in the literature. Prior assessments create prominence on the unit of computerization to categorize the processes. The prominence review comprises computational fundamentals. We differentiate the three ultimate methods:

- Geometric methodologies that attempt to discover a simplex covering the image data.
- Statistical computing methods that practice certain scientific morphology.
- Heuristic methods are not meticulously dignified underneath a hypothetical framework.

The quantity of spectral signatures to custom an HSI is generally anonymous. Most endmember induction procedures are computationally costly, owing to attain distributed implementations

that can aid them to be a realistic method for representative applications. GPUs are an economical way to acquire significant speed-ups.

3.2.1. Geometric Methods

Geometric approaches are non-parametric and non-statistical and explore the robustness between the LMM and the philosophy of convex collections. These methods depend on the hypothesis that pixel spectra exist in great dimensional measurements and LMM benefits by residing the endmembers at the extremes of this capacity aiding the assorted pixels to arise. The purpose is to define the location of the vertices in turn determining an enclosed surface with the least possible measurements, however, still encompassing every single pixel.

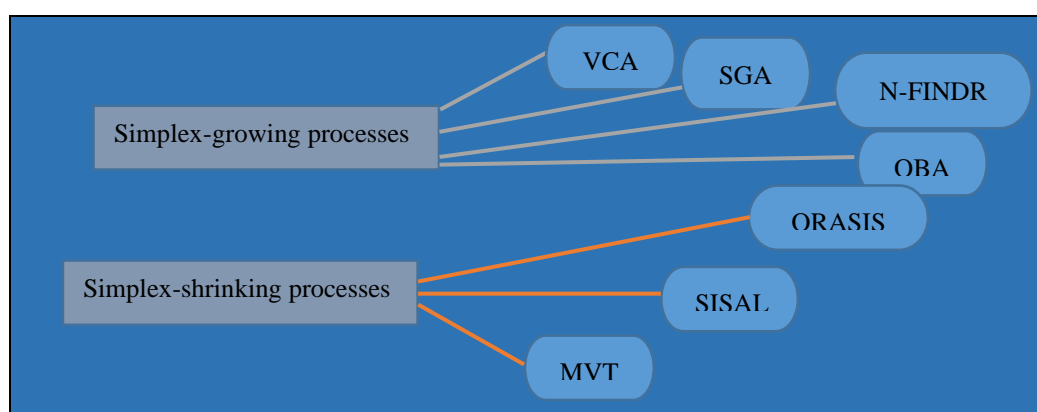


Fig. 4. The classification of geometric methods based on simplex shrinking or growing type [18]

The figure 4 determines the classification of geometric procedures centered on both simplex shrinking and growing type. These methods search for the apexes of a curving customary covering the illuminating statistics. Since the hyperspace data dissemination is generally tear-shaped, they seek the least possible simplex covering the entire statistics. The algorithms explore a preceded quantity of endmembers, demarcated by the user. The approaches reconnoiter the equivalence among mixing prototypes and the symmetrical alignment of HSI statistics in the multi-dimensional expanse. Undeviating spectral unmixing undertakes the composed continuums at the spectrometer and expresses it in the arrangement of a linear amalgamation of endmembers prejudiced by their consistent richness. This description convulsions well the geometrical assortment of endmembers across the vertices of a simplex, a convex, or a polyhedron conduit that nominally enfolds or is hugely enclosed in the statistics. The table 1 review the existing geometric centered procedures in that exists in the presence of linear data.

Table 1: Geometric based algorithms in the presence of linear data

| Algorithm | Methodology |
|------------------------------------|---|
| Minimum volume transform (MVT) [3] | Discover the least possible volume simplex embracing data cloud firmly. It presents twofold transforms, dark-point-fixed (DPF) transform, and fixed-point-free (FPF) transform. |

| | |
|--|--|
| <p>Convex Cone Analysis (CCA) [4]</p> | <p>The vectors molded by distinct illustrative spectra are lined amalgamations of nonnegative constituents and lie exclusively in a nonnegative convex area. CCA finds the boundaries of this convex region, using it as endmember spectra, performing a PCA provides DR on the illustration spectral association matrix. In this concentrated space, the endmembers essentially outline a convex cone on the encouraging hyperquadrant of the space, whose apex is in the space derivation.</p> |
| <p>N-FINDR [5] (It is a selection algorithm)</p> | <p>Finds pure pixels and uses them to describe the mixed pixels. Then discovers an internal simplex within the data and encloses the principal volume simplex. It begins with an indiscriminate pool of pixel spectra, analogous to the preliminary customary of endmembers. The residual illustrative pixels are a candidate to substitute respective endmember, if exploiting the dimensions, the simplex intensifies at that point it is acknowledged as the novel endmember and terminates once no substitutions are conceivable. It necessitates a DR phase, originally an OSP.</p> |
| <p>Simplex identification via split augmented lagrangian (SISAL) [8]</p> | <p>Estimates the least possible dimensions simplex to find delinquent as a categorization of convex optimization complications.</p> |
| <p>Pixel purity index (PPI) [9] (Endmember extraction algorithm)</p> | <p>It projects every pixel on a unique vector after a customary of unsystematic vectors traversing the illustrative space. A pixel obtains a score once it epitomizes an extremum of entire estimates. Pixels having uppermost scores are believed to be spectrally untainted.</p> |
| <p>The Iterated Constrained Endmembers (ICE) [13]</p> | <p>It accomplishes the minimization of a normalized remaining sum of squares. The normalization term is the dimensions of the simplex. The free parameters are the endmembers and the magnitudes for every pixel it iterates the solution of the tow interweaved and dependent minimization difficulties: leading the magnitudes, they are calculated by quadratic encoding delinquent solving, then the endmembers are computed as the direct minimization of the RSS functional. It does prerequisite a DR step, it is achieved by MNF algorithm. It is a progressive process to discover a simplex with the extreme capacity each time a fresh vertex is supplemented. VD is functional as discontinuing imperative to define the number of vertices mandatory.</p> |
| <p>Iterative constrained endmembers (ICE) with its sparsity-promoting version SPICE [14]</p> | <p>Accumulation of a sparsity endorsing term in the RSS leads to SPICE [6+]. This sparsity upholding term is an exchange of a Gaussian preceding by a Laplacian prior to the origination of the RSS. It permits the assortment of a suitable quantity of endmembers created on the sparsity quantity.</p> |

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| <p>Nonnegative matrix factorization (NMF)</p> | <p>Discover the convex polyhedron that superlatively outbursts the data and recognizes the vertices of the entity as the endmember. It does not require untainted pixels.</p> |
| <p>Minimum volume constrained nonnegative matrix factorization (MVC-NMF) [16]</p> | <p>Introducing a dimensions regularization term, in the MNF, appropriates the endmember abstraction for extremely assorted data. It earns the benefit of the profligate conjunction of NMF outlines and also eradicates the untainted pixel hypothesis. It entails the reformulation of an NMF cost-utility presenting a dimensions normalization term, substituting the RSS by the NMF standards.</p> |
| <p>Convex cone analysis (CCA) [21]</p> | <p>Builds a convex cone around the statistics beginning from an orthogonal base customary gained from a subgroup of the eigenvectors of the statistics association matrix. The quantity of base vectors is feedback to the process.</p> |
| <p>Simplex growing algorithm (SGA) [22]</p> | <p>SGA discovers extreme dimensions for an arrangement of steady rising simplexes vertex. It advances N-FINDR by comprising a method of rising simplex one vertex at an interval until the preferred quantity of vertices is gotten, declining the computational complexity. It selects a suitable initial vector to evade using unsystematic vectors as a primary state, yielding diverse arrays of absolute endmembers with diverse arrays of arbitrarily produced preliminary endmembers [22].</p> |
| <p>Vertex component analysis algorithm (VCA) [23] (unsupervised algorithm)</p> | <p>VCA repetitively executes orthogonal subspace analyses resultant from a structure of regular mounting simplexes to discover novel vertices. It repeatedly develops statistics onto a direction orthogonal to the subspace traversed by the endmembers previously resolute. The novel endmember signature relates to the extreme of the prognosis. It reiterates in anticipation that all endmembers are explored. Its mechanism includes projected as well as unprojected facts.</p> |
| <p>Orthogonal bases algorithm (OBA) [23]</p> | <p>Abstracts endmembers successively by computations of major simplex dimensions. Then replace simplex with the major dimensions by computing a factor with examining for a novel orthogonal base having the principal model and is the resulting endmember.</p> |
| <p>Sequential maximum angle convex cone (SMACC) [23] sequential algorithm</p> | <p>Develops a convex cone as an alternative to a simplex. A novel endmember is recognized created on the viewpoint it creates by the present convex cone. The statistics vector creating a supreme perspective with the existing convex cone is selected as the subsequent endmember to enhance to expand the endmember customary. Profusion maps are simultaneously created and rationalized at every stage.</p> |

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| Iterative error analysis (IEA) [21] (sequential method) | Depend on the presence of untainted pixels. A series of linear inhibited HU is accomplished by electing as endmembers the pixels that minimalize the residual inaccuracy in the authentic image. |
|---|--|

The figure 5 shows the various categories in the Nonlinear Geometric algorithms.

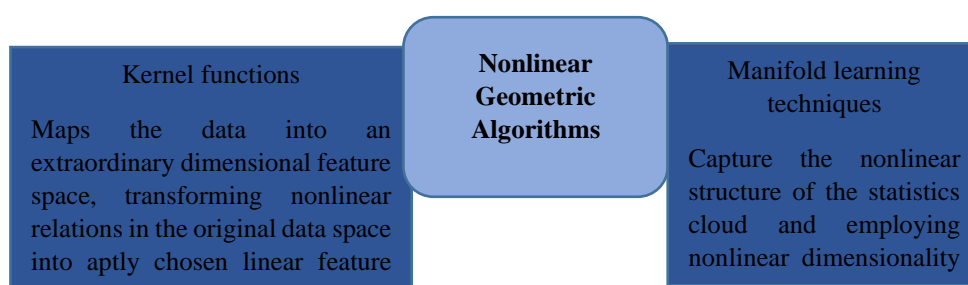


Fig. 5. Types of Nonlinear Geometric algorithms [21]

3.2.2. Statistical Methods

Statistical methods optimize the objective functions derived from the data statistics to recognize endmembers. Besides, the non-parametric procedures endeavor deprived of enhancing a parametric archetypal, nonetheless minimize the objective utility by the statistics derived from the data. It is demarcated as the assortment of computational approaches that are either definite on the algebra of static operatives or engage its philosophy to simplify earlier methods. Scientific Morphology is an appropriate case of this archetype, however, it similarly embraces certain fuzzy structures methods and NNs. If a HU process method and assorted pixel with statistical illustrations, then the process is statistical. The representations can be analytical expressions representing probability density function. Automated morphological endmember abstraction is one such process that is created on the characterization of multispectral attrition and expansion operatives, which are formerly utilized to calculate the Morphological Eccentricity Index (MEI) over kernels of a cumulative extent that are calculated over all the pixels. The outcome is an MEI image with maxima corresponding to the endmember pixels. The process does not requisite dimensionality reduction. The table 2 illustrates the analysis of existing statistical algorithms and their principle of functioning.

Table 2: Statistical Algorithms and their functioning principle

| Method | Functioning Principle |
|----------------------------------|--|
| Spatial pre processing (SPP) [8] | Used in grouping with a prevailing (geometrical or statistical) process. |
| Hierarchical Bayesian model [4] | Every pixel of the HSI is putrid as a linear arrangement of untainted endmember bands. The estimate is led by creating the posterior dispersal of profusions and endmember |

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| | constraints. It provides dual endmember mining and profusion assessment for HSI. |
| ICA-based abundance quantification algorithm (ICA-AQA) [5] | A high directive data method was established to achieve endmember abstraction and profusion quantification concurrently. Certain nonparametric statistical HU approaches propose deviations on it. |
| Dependent component analysis (DECA) [15] | Models the abundance fractions as assortments of Dirichlet bulk, thus imposing the restraints on abundance segments enforced by the attainment process, called non-negativity, and has a persistent sum. |
| Spatial-spectral endmember extraction algorithm (SSEE) [23] | A mechanism is evaluating a part in fragments increasing the spectral disparity of truncated disparity endmembers, thus refining the probable endmembers to be selected. |
| Markov random field (MRF) [31] | Defines a panel of spectrally assorted pixels into spatially consistent areas. |

The notion of morphological impartiality, reformulated as static independence, is the elementary tool in the approach [17]. The customary of endmembers is framed as a customary of morphologically autonomous vectors. There the Associative Morphological Memories, are projected as indicators of morphologically sovereign vectors. The process mechanism is a solitary authorization over the illustration statistics. This method is trailed in [26]. The association between robust lattice individuality and affine impartiality was demonstrated and was found that most vectors in the erosive and dilative lattice memories are robust lattice autonomous. Consequently, the mere creation of the lattice remembrances offers an approach to attain the convex hull of the statistics. Providing an endmember assortment mechanism, the process can acquire endmembers customary in a single authorization. The figure 6 determines the additional statistical categories of algorithms.

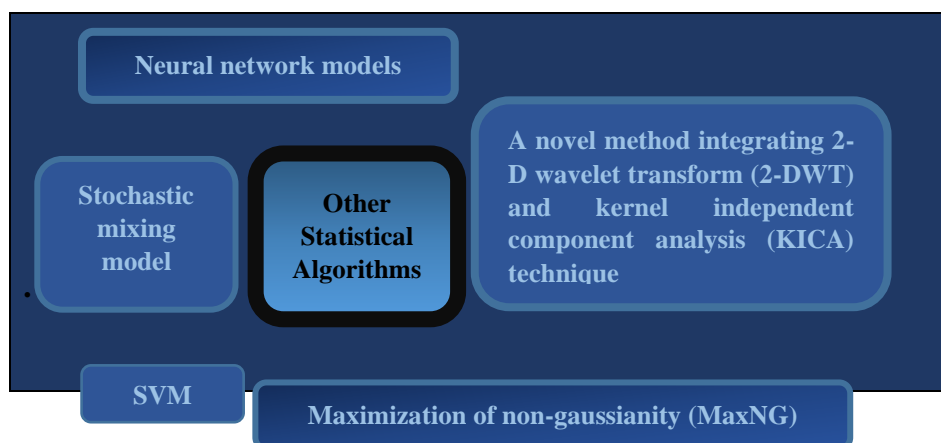


Fig. 6. Additional statistical types of algorithms [26]

3.2.2.1. Nonnegative Matrix Factorization (NMF)

NMF is well adapted for face analysis and clustering of documents, the objective function is non-convex that results in a vast solution space. Various extension techniques have been projected by engaging appropriate priors to confine the resolution space. For the HU task, the priors are enforced either on an abundance of endmembers.

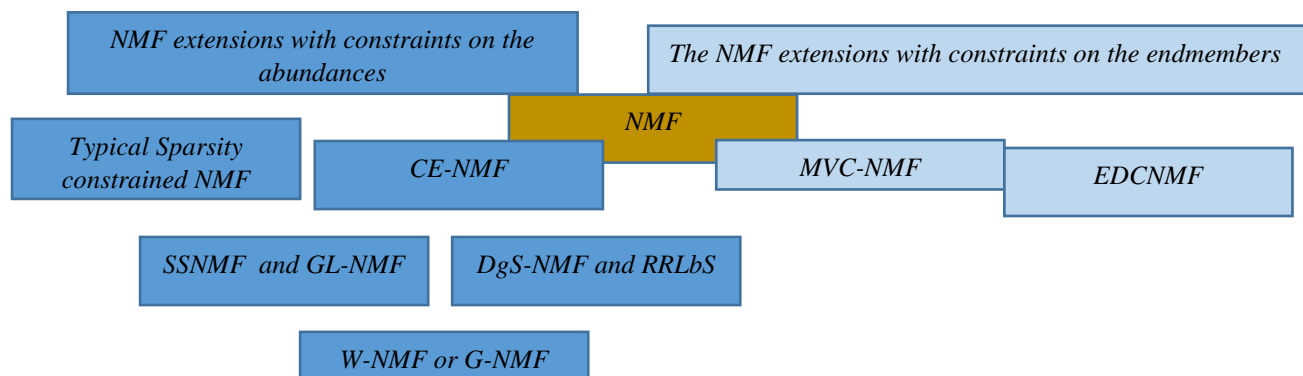


Fig. 7. Categorization of NMF based on an extension in either endmember or abundance [27]

a. The NMF extensions with the abundance constraints [27]:

The native neighborhood weight normalized NMF (W-NMF) undertakes that HSI pixels are distributed on an assorted; exploiting suitable weights in the local neighborhood enhancing the spectral unmixing. Precisely, W-NMF employs both the spectral besides spatial information to create the weight matrix. The sparsity-constrained NMFs [28] are the most effective approaches for the HU task. The figure 7 depicts the categorization of NMF based on an extension in either endmember or abundance. Those methods assume that most hyperspectral pixels are mixed with parts of endmembers, and exploit all types of sparse constraints on the abundance. To condense the resolution space, existing HU methodologies exploit several constraints on the abundances as well as on the endmembers. Nevertheless, they employ an indistinguishable strength of restraints on every factor, not meeting the practical situation. Instead, [9] perceived that the assorted level of every pixel diverges concluding image grids. Established on this prior, a unique method is proposed to acquire a data-guided map (Dg Map) that aims to define the assorted level of every pixel. DgS-NMF [9] is an interesting technique. However, a heuristic procedure is proposed in [9] to learn the Dg Map, being ineffective for the massive smooth regions in the image. It is projected that a more precise Dg Map constraint would bias the result to additional acceptable local minima. Moreover, the existing methods usually overlook the critically degraded channels in the HSI. Addressing the above two problems, a vigorous illustration and erudition-based sparsity process is anticipated by accentuating both vigorous demonstration and erudition-based sparsity. There are two methods considering both the spatial (like a graph) constraint and the sparsity constraint. In the Structured (or Graph) sparsed NMF (SS-NMF), the constraint is the graph Laplacian L is erudite via a unique way that considers the spectral and spatial information in the HSI. It is identified that Euclidean loss is prone to deviates. For the Correntropy-based NMF (CE-NMF),

[28] the correntropy metric is engaged in measuring the error of reconstruction that results in a new robust objective.

b. The NMF extensions with endmember constraints [6]:

The Minimum Volume Constrained NMF (MVC-NMF) combines the property of both the geometric and statistical approaches. Its goals are to discover the endmembers, which compose the minimum volume simplex that demarcates the HSI data scatters. Inspired by the MVC-NMF, the Endmember Dissimilarity Constrained NMF (EDCNMF) is proposed [29]. The primary hypothesis is that owing to the extraordinary spectral determination of HSI sensors, the endmember spectra must be smooth and diverse from each other.

3.2.3. Heuristic Methods

The heuristic approaches accumulate a set of assorted endmember abstraction approaches that practice diverse methods not clustered beneath a firm theatrical contextual for endmember orientation. The table 3 highlights the major heuristic algorithms along with their purposes and advantages.

Table 3: Heuristic Algorithms with its purpose and advantage

| Algorithm | Purpose | Advantage |
|---|--|---|
| ICA and Independent Factor Analysis (IFA) [7] | It shows that the statistically sovereign of the foundations, anticipated by ICA fused IFA, is disrupted in HU, compromising the recital of the processes for this resolution. | The accurateness of the procedures inclines to advance the upsurge of the signature erraticism and the SNR. |
| Spatial-Spectral Endmember Extraction (SSEE) [19] | It uses singular value putrefaction to regulate a customary source vector to define most of the spectral discrepancy for subgroups of the image. Then the complete image dataset is anticipated onto a locally definite vector to define an agreed candidate endmember pixel from where the concluding endmembers are designated. For that, it examines spectrally alike but spatially liberated endmembers. | It is a prognosis-based scheme with a mechanism that analyses a scene in fragments and increases the spectral disparity of truncated contrast endmembers, thus refining the probable endmembers to be designated. |
| Pixel Purity Index (PPI) [30]. | It lessens statistics dimensionality and produces a noise whitened procedure by MNF process, and regulates the pixel pureness by repetitively prominent statistics onto arbitrary element vectors. The extreme pixel in every projection is calculated, the outcome the untainted | The most eminent and extensively used process, owing to its enclosure in the ENVI software package. Although PPI is intensively used, its execution facets are |

| | | |
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| | pixels. PPI desires human interference to select the extreme pixels that finest gratify the target spectrum. | kept indefinite owing to the inadequate consequences. |
| The Fast Iterative PPI (FIPPI) [30] | FIPPI yields a suitable preliminary customary of endmembers to rapidly up the procedure. Furthermore, it evaluates the number of endmembers to be created by VD. It is an unsupervised and reiterative process, with an iterative regulation to recover every reiteration till it affects a concluding agreed endmember. | It improves PPI in several aspects. |
| ICA-based Abundance Quantification (ICA-AQA) [31] | It is an extraordinary statistics-centered method to achieve endmember abstraction and profusion quantification concurrently in the solitary shot procedure. | It is a virtuous method for endmember abstraction and profusion quantification. |

3.2.4. Geometric Vs. Static

Geometric methods assume the existence of deterministic endmembers at the simplex vertices, subsequently, remain sensitive towards deviates and bad pixels owing to defective essentials on the sensor focal-plane array. Yet, geometric methods expose erratic objects that are frequently overlooked by statistical methodologies. As a result, making them proficient to expose low likelihood targets. However, a disadvantage, is that anomalous pixels arising from sensor artifacts are also acknowledged as endmembers. Accordingly, geometric methods are specifically suitable to recognize truncated possibility targets, however, performing preeminent when data is free of artifacts that in turn generate spurious results. Geometric methods are the finest one hypothetically, but they require pronounced computational resources, and also the endmembers obtained have no vibrant physical meaning.

4. Inversion Taxonomy

This section addresses inversion, yielding the second principal output of HU. The processes aim is to define the insignificant existence of every endmember in the acknowledged pixel spectrum. The perplexing aspect of inversion is defining the merging of scientific and statistical techniques with the underlying physical limitations. Henceforth, any approximation of abundance essentially conforms to the restraints of non-negativity, pureness, and complete or partial additive. Numerous endmember-determination procedures simultaneously estimate the abundance of endmembers. In a scene, often, both the endmembers and profusions are indefinite, and usually, both measures are pursued concurrently. However, there are circumstances, when the endmembers are acknowledged and the merely undertaking is recovering the profusions.

4.1. Inversion Algorithms

These are ruled by methods that raise certain aspects of the least-squares method. Several statistical and parametric algorithms perform least-squares exploration either by the squared-error minutest criteria. The prevalence of inversion systems grounded on least-squares validates the contemporary expansion of HSI algorithms, whether appropriately or not, on a single belief of expanse. Clustering algorithms used to identify endmember often yield abundance estimates as well. Several methodologies discussed previously for endmember delineate also assess profusions concurrently. For mixed pixels, the presence of fractional abundances of endmembers is the prime product of HU. The pixel spectrum is frequently exhibited as a linear amalgamation of endmember spectra biased by their equivalent profusions, and the abundances are resultant by the least-squares process to lessen the mean square error among the definite spectrum and the recreated spectrum. This method despite its mathematic ease approximates the profusions in a pixel-by-pixel manner and for every pixel, the profusions are imitative autonomously of its adjacent pixels. Nevertheless, the amalgamation of spatial data improves the precision of profusion approximation and emboldens resultant profusions to be further spatially reliable. For each pixel, a sequence of spatial measures demarcated on the RMSE was engaged for an iterative HU technique that aims to remove spatial erection lasting in the residuals. Still, spatial measures were not encompassed in an objective utility to be improved. In [29], the pixel profusion is anticipated through the weighted average of the abundances in the neighboring pixels and the objective utility is created to exploit the spatial certainty of the profusions. For spectral unmixing, MRF was also exploited. Implicit image classification is accompanied in the projected method to divide the image into consistent areas with the statistical moments of fractional profusions unaffected. To deduce the dispersals of the abundances and class labels, a hierarchical Bayesian model is embraced. Furthermore, local homogeneity is considered for selecting the landmark points in the nonlinear manifold-based spectral unmixing.

Conclusions

The arena of HSI processing is an application field for several methods. Amongst them, HU offers a form of somatic image archetypal with simple elucidation allowing subpixel resolution outcomes. The paper provides a hierarchically organized structure of algorithms existing in the literature for spectral unmixing. In HSI analysis, spectral unmixing is a tool. For this analysis, a requisite is the endmember's determination. In this review paper, the existing approaches support the endmember orientation from the image statistics. The endmembers are anticipated to have specific physical significance, probably in the case of methodologies that achieve an assortment from the illumination pixel spectra. Though the methods typically yield convex polytopes to cover altogether the points in image statistics, therefore, the candidate customary of endmembers does not adequate in the official explanation of endmembers. Moreover, to increase the performance of HU, spatial information incorporation has achieved great success. The paper gives a brief of the existing spatial-spectral HU methods in the three phases: endmember abstraction, an assortment of endmember amalgamations, and profusion estimation. The area of spatial-spectral HU is less recognized and entails additional attention

in the field of HSI unmixing. Also, organizations of the paper are created from the perspective that HU is an inverse delinquent that endeavors to evaluate imperative physical constraints from illumination interacting with the material of concentration. Complete information on each variable is unrealistic, and this detail is exhibited in fluctuating units by the norms enforced by processes on the unmixing complications. The categorizations reveal the inclusive disparity existing in the approaches. Hyperspectral unmixing illustrates an area that has raised a diverse assembly of participants. Consolidating and categorizing these approaches is a progressively imperative task and it is expected that forthcoming nomenclatures established for HU will reveal the growing superiority of a field. This review also promotes the advance of the assimilation of spatial data in HU. In the future, an exhaustive quantifiable and proportional assessment can be provided for certain appropriate approaches defined in this survey, exhausting surplus evaluation metrics.

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